Day 28 Sept 2020 Ex. 8.1

Q1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the Quadrilateral.

Sol. Let the Common ratio between the angles be x.
We know that the sum of the interior angles of the quadrilateral = 360°

Now ⇒ $3x + 5x + 9x + 13x = 360°$

$30x = 360°$

$x = \frac{360}{30} = 12°$

Angles of the quadrilateral are:-

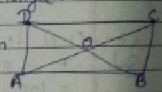
$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

$13x = 13 \times 12 = 156°$

Q2. If the diagonals of a parallelogram are equal then show that it is a rectangle.



Sol. Let us draw ABCD parallelogram in which AC and BD are two equal diagonals.

$\angle ABC + \angle ABC = 180°$

$2\angle ABC = 180°$

$\angle ABC = \frac{180}{2} = 90°$

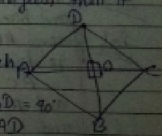
AB = DC (opp. side of ||gram are equal)

$\triangle ABC \cong \triangle DCB$ (By SSS Rule)

$\angle ABC = \angle DCB$ (CPCT)

$\angle ABC + \angle DCB = 180°$ (Co interior angle)

Q3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Sol. Let us draw a Quad. ABCD in which

OA = OC, OB = OD

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$

To Prove :) AB = BC = CD = AD

In $\triangle AOB$ & $\triangle AOD$

OA = OA (common side)

OB = OD (Given)

$\angle AOB = \angle AOD$ (90°)

$\triangle AOB \cong \triangle AOD$

∴ AB = AD (By CPCT)

In $\triangle AOB$ and $\triangle BOC$

OB = OB (Common side)

OA = OC (Given)

$\angle AOB = \angle BOC$ (By 90°)

$\triangle AOB \cong \triangle BOC$

∴ AB = BC (By CPCT)

In $\triangle BOC$ & $\triangle COD$

OC = OC (common side)

OB = OD (Given)

$\angle BOC = \angle COD$ (By 90°) $\triangle BOC \cong \triangle COD$

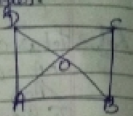
BC = CD (By CPCT)

AB = AD = BC = CD

AD = BC = CD = AD (H.P)

Then ABCD is a Rhombus.

Q4. Show that the diagonals of a square are equal and bisect each other at right angles.



Sol. Given: ABCD is a Square

To prove

$AC = BD$

and $AO = CO$

and $BO = DO$

Proof: In $\triangle ABD$ and $\triangle BAC$

$AB = BA$ (common)

$\angle BAD = \angle ABC$ (By 90°)

$AD = BC$ (Sides of Squares)

$\triangle ABD \cong \triangle BAC$ (By SAS Rule)

$BD = AC$ (By CPCT)

In $\triangle AOB$ and $\triangle COD$

$\angle 1 = \angle 3$ (Alternate Interior angle)

$AB = CD$ (sides of square)

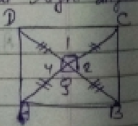
$\angle 2 = \angle 4$ (Alternate Interior angle)

$\triangle AOB \cong \triangle COD$ (By ASA Rule)

$AO = CO$ (By CPCT)

$OB = OD$ (By CPCT)

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a Square.



Sol. Given $AC = BD$

$AO = CO$ and $BO = DO$

$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90°$

To prove ABCD is a Square

Proof In $\triangle AOB$ and $\triangle COB$

$AO = CO$ (given)

$\angle 1 = \angle 2$ (given)

$OB = OB$ (common)

$\triangle AOB \cong \triangle COB$ (By SAS Rule)

$AB = CB$ (By CPCT) ——— (i)

Same as:-

$\triangle BOC \cong \triangle DOC$

$CB = CD$ ——— (ii) (By CPCT)

Same as:-

$\triangle COD \cong \triangle AOD$

$CD = AD$ ——— (iii) (By CPCT)

from eq. (i) (ii) and (iii)

$AB = CB = CD = AD$

ABCD is rhombus

In $\triangle ABC$ and $\triangle BAD$

$AB = BA$ (common)

$BC = AD$ (ABCD is a rhombus)

$AC = BD$ (Given)

$\therefore \triangle ABC \cong \triangle BAD$ (By SSS)

$\angle A = \angle B$ (By CPCT)

$\angle A + \angle B = 180°$ ($AD \parallel BC$)

$\angle A + \angle A = 180°$ ($\angle A = \angle B$)

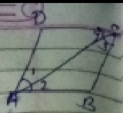
$2\angle A = 180°$

$\angle A = \frac{180}{2} = 90°$

Q6. Diagonal AC of a parallelogram ABCD bisects $\angle A$. Show that.

(i) it bisects $\angle C$ also.

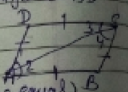
(ii) ABCD is a rhombus.



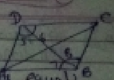
Sol. (i) $\angle 1 = \angle 2$ (AC Bisect $\angle A$) — ①
$\angle 2 = \angle 4$ (Alternate Interior Angle) — ②
$\angle 1 = \angle 3$ (AIA) — ③
By eq. ① ② ③
$\angle 1 = \angle 2 = \angle 3 = \angle 4$ — ④
$\angle 3 = \angle 4$
∴ AC bisect $\angle C$ Also.
(ii) $\angle 1 = \angle 2 = \angle 3 = \angle 4$
$CD = BC = AB = AD$ (side opp. to equal angles in Δ are eq.)

$AB = BC = CD = AD$
Then ABCD is Rhombus.

Q7. ABCD is a rhombus, Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

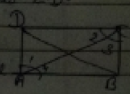


Sol In ΔABC
$AB = BC$ (All side of Rhombus are equal)
$\angle 4 = \angle 2$ (Angle opp. equal are equal) — ①
$\angle 2 = \angle 3$ (AIA) — ②
$\angle 4 = \angle 2 = \angle 3$
$\angle 4 = \angle 3$ (eq. (i) & (ii))
$\angle 4 = \angle 1$ (AIA — (iii)
$\angle 4 = \angle 2$
$\angle 2 = \angle 1$
Therefore AC Bisect $\angle A$ and $\angle C$.



(ii) In ΔABD
$AB = AD$ (side of Rhombus always equal)
$\angle 5 = \angle 7$ (Angle opp. to equal side are equal) — (iv)
$\angle 7 = \angle 6$ (AIA) — (v)
$\angle 5 = \angle 8$ (AIA) — (vi)
$\angle 5 = \angle 6$ (By eq. (iv) & (v))
$\angle 8 = \angle 7$ (By eq. (iv) & vi)
Therefore BD bisect $\angle B$ and $\angle D$.

Q8. ABCD is a rectangle in which diagonal AC $\angle A$ as well as $\angle C$. Show that
(i) ABCD is a square.
(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.



Sol. $\angle 1 = \angle 2$
In ΔACD [$\angle 1 = \angle 3$, $\angle 3 = \angle 2$]
$CD = AD$ (side opp. to equal angle are equal) — ①
$AB = CD$ — ②
$AD = BC$ — ③
$AB = CD = AD = BC$
$AB = BC = CD = AD$ (By eq. ①, ② and ③)
∴ ABCD is a Square.
$AB = BC = CD = AD$ — ④
$BC = CD$
$\angle 5 = \angle 6$ (Angle opp. to equal sides are equal) — ⑤
$\angle 5 = \angle 7$ [AIA (Alternat Int. Angle) — ⑥
$\angle 6 = \angle 8$ [AIA — ⑦
$\angle 6 = \angle 7$
$\angle 5 = \angle 8$ [BD bisect $\angle B$ as well as $\angle D$.

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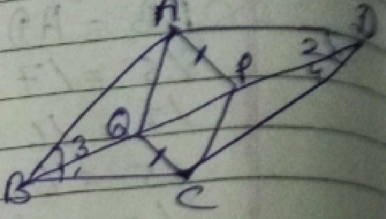
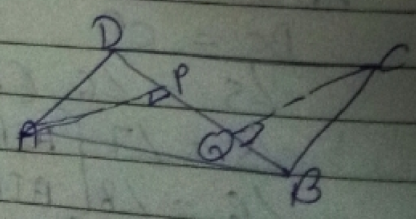
Q9. In parallelogram ABCD, two points P & Q are taken on diagonal BD such that DP = BQ. Show that:



Sol. Given ABCD is a ∥gram
and DP = BQ
To Show (i) ΔAPD ≅ ΔCQB
(ii) AP = CQ
Proof in In ΔAPD and ΔCQB
DP = BQ (given)
∠2 = ∠1 (IAA)
AD = CB (opp. sides of ∥gram.)
ΔAPD ≅ ΔCQB (By SAS Rule)
AP = CQ (By CPCT) — (1)
In ΔAQB and ΔCPD
QB = PD (given
∠3 = ∠4 (IAA)
AB = CD (opp. sides of ∥gram)
ΔAQB ≅ ΔCPD (By SAS Rule)
AQ = CP (By CPCT) — (2)
from eq (1) & (2)
APCQ is a Prallelogram

Q10. ABCD is a ∥gram and AP and CQ are ⊥ from vertices A and C on diagonal BD. Show that
(i) ΔAPB ≅ ΔCQD
(ii) AP = CQ



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Sol. In ΔAPB and ΔCQD

$AB = CD$ (opp. side of ∥gram)

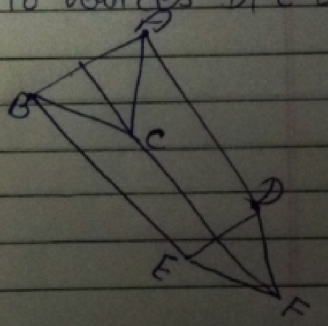
$\angle APB = \angle CQD$ (By 90°) AP and CQ are ⊥)

$\angle ABP = \angle CDQ$ (AIA

$\Delta APB \cong \Delta CQD$ (SAA)

$AP = CQ$ (By CPCT)

Q11. In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and BC [∥ EF]. Vertices A, B and C are joined to vertices D, E & F respectively. Show that.



(i) Quad. ABED is ∥gram

(ii) Quad. BEFC is ∥gram

(iii) Quad. ACFD is ∥gram

(iv) $AD \parallel CF$ and $AD = CF$

(v) $AC = DF$

(vi) $\Delta ABC \cong \Delta DEF$

Solution 11:

(i) It is given that AB = DE and AB || DE.

If two opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram.

(ii) Again, BC = EF and BC || EF

Therefore, quadrilateral BCEF is a parallelogram.

(iii) As we had observed that ABED and BEFC are parallelograms, therefore

AD = BE and AD || BE

(Opposite sides of a parallelogram are equal and parallel)

And, BE = CF and BE || CF

(Opposite sides of a parallelogram are equal and parallel)

∴ AD = CF and AD || CF

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral ACFD are equal and parallel to each other, therefore, it is a parallelogram.

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.

∴ AC || DF and AC = DF

(vi) ΔABC and ΔDEF,

AB = DE (Given)

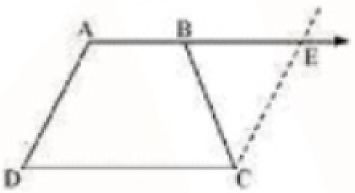
BC = EF (Given)

AC = DF (ACFD is a parallelogram)

∴ ΔABC ≅ ΔDEF (By SSS congruence rule)

Question 12:

ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that



(i) ∠A = ∠B

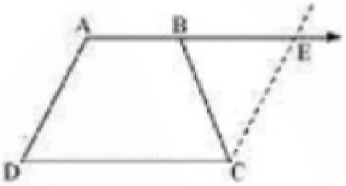
(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

[redacted]

Question 12:

ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that



(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Solution 12:

Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E. It is clear that AECD is a parallelogram.

(i) AD = CE (Opposite sides of parallelogram AECD)

However, AD = BC (Given)

Therefore, BC = CE

∠CEB = ∠CBE (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them.

∠A + ∠CEB = 180° (Angles on the same side of transversal)

∠A + ∠CBE = 180° (Using the relation ∠CEB = ∠CBE) ... (1)

However, ∠B + ∠CBE = 180° (Linear pair angles) ... (2)

From Equations (1) and (2), we obtain

[redacted]

∠A = ∠B

(ii) AB ∥ CD

∠A + ∠D = 180° (Angles on the same side of the transversal)

Also, ∠C + ∠B = 180° (Angles on the same side of the transversal)

∴ ∠A + ∠D = ∠C + ∠B

However, ∠A = ∠B [Using the result obtained in (i)]

∴ ∠C = ∠D

(iii) In ΔABC and ΔBAD,

AB = BA (Common side)

BC = AD (Given)

∠B = ∠A (Proved before)

∴ ΔABC ≅ ΔBAD (SAS congruence rule)

(iv) We had observed that,

ΔABC ≅ ΔBAD

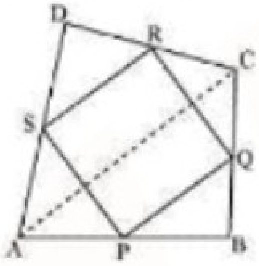
∴ AC = BD (By CPCT)

[redacted]

Exercise (8.2)

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Solution 1:

(i) In ΔADC, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC ... (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$AC ... (2)

Using Equations (1) and (2), we obtain

PQ || SR and PQ = SR ... (3)

$\therefore$ PQ = SR

(iii) From Equation (3), we obtained

PQ || SR and PQ = SR

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

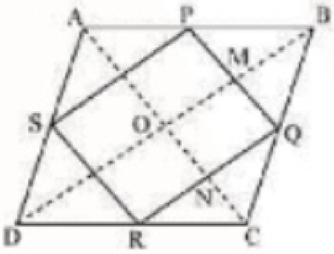
[redacted]

Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution 2:



In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ (Using mid-point theorem) ... (1)

In $\triangle ADC$,

R and S are the mid-points of CD and AD respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ (Using mid-point theorem) ... (2)

From Equations (1) and (2), we obtain

$PQ \parallel RS$ and $PQ = RS$

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

$MQ \parallel ON$ ($\because PQ \parallel AC$)

[redacted]

QN ∥ OM (∵ QR ∥ BD)

Therefore, OMQN is a parallelogram.

∴ ∠MQN = ∠NOM

∴ ∠PQR = ∠NOM

However, ∠NOM = 90° (Diagonals of a rhombus are perpendicular to each other)

∴ ∠PQR = 90°

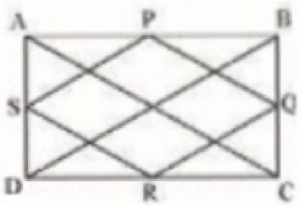
Clearly, PQRS is a parallelogram having one of its interior angles as 90°.

Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution 3:



Let us join AC and BD.

In ΔABC,

P and Q are the mid-points of AB and BC respectively.

∴ PQ ∥ AC and PQ = $\frac{1}{2}$AC (Mid-point theorem) ... (1)

Similarly in ΔADC,

[redacted]

SR || AC and SR = $\frac{1}{2}$AC (Mid-point theorem) ... (2)

Clearly, PQ || SR and PQ = SR

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

∴ PS || QR and PS = QR (Opposite sides of parallelogram) ... (3)

In ΔBCD, Q and R are the mid-points of side BC and CD respectively.

∴ QR || BD and QR = $\frac{1}{2}$BD (Mid-point theorem) ... (4)

However, the diagonals of a rectangle are equal.

∴ AC = BD ...(5)

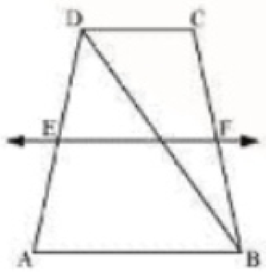
By using Equations (1), (2), (3), (4), and (5), we obtain

PQ = QR = SR = PS

Therefore, PQRS is a rhombus.

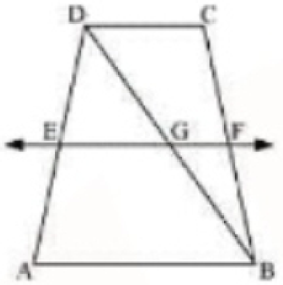
Question 4:

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid - point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.



Solution 4:

Let EF intersect DB at G.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

In ΔABD,

EF || AB and E is the mid-point of AD.

Therefore, G will be the mid-point of DB.

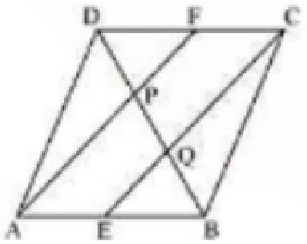
As EF || AB and AB || CD,

∴ EF || CD (Two lines parallel to the same line are parallel to each other)

In ΔBCD, GF || CD and G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC.

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



ABCD is a parallelogram.

AB || CD

And hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}$AB = $\frac{1}{2}$CD

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

∴ AF || EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

∴ DP = PQ ... (1)

Similarly, in ΔAPB, E is the mid-point of side AB and EQ || AP (as AF || EC).

Therefore, by using the converse of mid-point theorem, it can be said that Q is the mid-point of PB.

∴ PQ = QB ... (2)

From Equations (1) and (2),

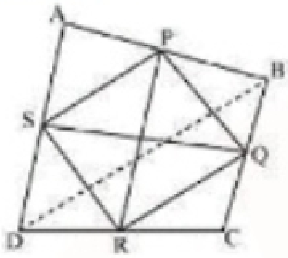
DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution 6:



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.

In ΔABD, S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ ... (1)

Similarly in ΔBCD,

$QR \parallel BD$ and $QR = \frac{1}{2}BD$ ... (2)

From Equations (1) and (2), we obtain

$SP \parallel QR$ and $SP = QR$

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Therefore, SPQR is a parallelogram.

We know that diagonals of a parallelogram bisect each other.

Hence, PR and QS bisect each other.

Question 7:

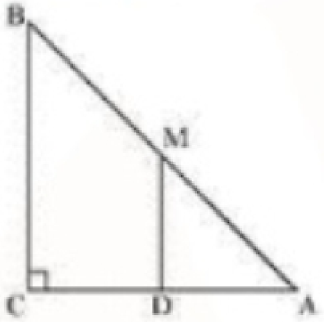
ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2} AB$

Solution 7:



(i) In ΔABC,

It is given that M is the mid-point of AB and MD || BC.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As DM || CB and AC is a transversal line for them, therefore,

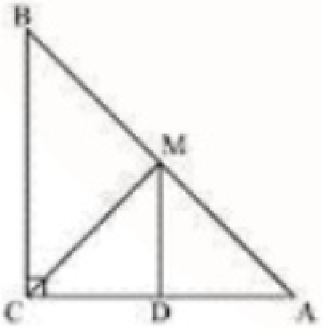
$\angle MDC + \angle DCB = 180^\circ$ (Co-interior angles)

$\angle MDC + 90^\circ = 180^\circ$

$\angle MDC = 90^\circ$

$\therefore MD \perp AC$

(iii) Join MC.



In ΔAMD and ΔCMD,

$AD = CD$ (D is the mid-point of side AC)

$\angle ADM = \angle CDM$ (Each 90°)

DM = DM (Common)

∴ $\Delta AMD \cong \Delta CMD$ (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

However, AM = $\frac{1}{2}$AB (M is the mid-point of AB)

Therefore, it can be said that

CM = AM = $\frac{1}{2}$AB